Bipartisan Paxos: A Modular State Machine Replication Protocol

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Abstract

There is no shortage of state machine replication protocols. From Generalized Paxos to EPaxos, a huge number of replication protocols have been proposed that achieve high throughput *and* low latency. However, these protocols all have two problems. First, they do not scale. Many protocols actually slow down when you scale them, instead of speeding up. For example, increasing the number of MultiPaxos acceptors increases quorum sizes and slows down the protocol. Second, they are too complicated. This is not a secret; state machine replication is notoriously difficult to understand.

In this paper, we tackle both problems with a single solution: *modularity*. We present Bipartisan Paxos (BPaxos), a modular state machine replication protocol. Modularity yields high throughput via scaling. We note that while many replication protocol components do not scale, some do. By modularizing BPaxos, we are able to disentangle the two and scale the bottleneck components to increase the protocol's throughput. Modularity also yields simplicity. BPaxos is divided into a number of independent modules that can be understood and proven correct in isolation.

1 Introduction

State machine replication protocols like MultiPaxos [12, 15] and Raft [25] allow a state machine to be executed in unison across a number of machines, despite the possibility of faults. Today, state machine replication is pervasive. Nearly every strongly consistent distributed system is implemented with some form of state machine replication [1,2,4,5,7,9,10,32].

MultiPaxos is one of the oldest and one of the most widely used state machine replication protocols. However, despite its popularity, MultiPaxos does not have optimal throughput or optimal latency. In response, a number of state machine replication protocols have been proposed to address MultiPaxos' suboptimal performance [6, 8, 11, 13, 14, 17, 20, 23, 24, 26, 27]. These protocols use sophisticated techniques that either increase MultiPaxos' throughput, decrease its latency, or both

These sophisticated replication protocols have two shortcomings: they do not scale, and they are very complex. In this paper, we address both these shortcomings with a single solution: **modularity**. We present Bipartisan Paxos (BPaxos), a state machine replication protocol that is composed of a number of independent modules. Modularity allows us to achieve state-of-the-art throughput via a straightforward form of scaling. Furthermore, modularity makes BPaxos significantly easier to understand compared to similar protocols. **Scaling** Simple state machine replication protocols like MultiPaxos and Raft cannot take advantage of scaling. Conventional wisdom encourages us to use as few nodes as possible when deploying these protocols: "using more than 2f + 1 replicas for f failures is possible but illogical because it requires a larger quorum size with no additional benefit" [34]. While some protocols use multiple leaders [6,20,23], the number of leaders is fixed (typically 2f + 1 leaders to tolerate f faults), which only alleviates but does not solve the scalability problem.

BPaxos, on the other hand, employs a straightforward form of scaling to achieve high throughput. A BPaxos deployment consists of a set of leaders, dependency service nodes. proposers, acceptors, and replicas. We will see later that dependency service nodes, acceptors, and replicas do not scale. This is why conventional wisdom dictates using as few of these nodes as possible. However, leaders and proposers operate independently from one another and are thus "embarrassingly scalable". Moreover, when we analyze the performance of BPaxos, we find that these leaders and proposers are the throughput bottleneck. By increasing the number of leaders and proposers, we increase the protocol's throughput. Note that BPaxos does not horizontally scale forever. Scaling the leaders and proposers shifts the bottleneck to other non-scalable components. With scaling, BPaxos is able to achieve roughly double the peak throughput of EPaxos, a state-of-the-art replication protocol.

This straightforward form of scaling has been largely overlooked because most existing replication protocols tightly couple their components together. For example, an EPaxos replica plays the role of a leader, a dependency service node, an acceptor, *and* a replica [6]. This tight coupling has a number of advantages—e.g., messages sent between co-located nodes do not have to traverse the network, redundant metadata can be coalesced, fast paths can be taken to reduce latency, and so on. However, tight coupling lumps together components that do not scale with components that do. This prevents independently scaling bottleneck components. BPaxos' modularity is the key enabling feature that allows us to perform independent scaling.

Simplicity MultiPaxos is notoriously difficult to understand, and sophisticated protocols that improve it are significantly more complex. BPaxos' modular design, on the other hand, makes the protocol much easier to understand compared to these sophisticated protocols. Each module can be understood and proven correct in isolation, allowing newcomers to understand the protocol piece by piece, something that is difficult to do with existing protocols in which components are tightly coupled.

Moreover, some of the modules implement well-known abstractions for which well-established protocols already exist. In these cases, BPaxos can leverage existing protocols instead of reinventing the wheel. For example, BPaxos depends on a module that implements consensus. Rather than implementing a consensus protocol from scratch and having to prove it correct, BPaxos uses Paxos off the shelf and inherits its safety properties. Many other protocols [6,23,24] instead implement consensus in a way that is specialized to each protocol. These specialized consensus protocols are difficult to understand and difficult to prove correct. As an anecdote, we discovered a minor bug in EPaxos' implementation of consensus, which we confirmed with the authors, a bug that went undiscovered for six years.

Summary In summary, we present the following contributions:

- We introduce BPaxos, a modular, multileader, generalized state machine replication protocol that is significantly easier to understand compared to similar protocols.
- We describe how modularity enables a straightforward form of protocol scaling. We apply the technique to BPaxos and achieve double the peak throughput of a state-of-the-art replication protocol.

2 Background

2.1 Paxos

Assume we have a number of clients, each with a value that they would like to propose. The **consensus** problem is for all members to agree on a single value among the proposed values. A consensus protocol is a protocol that implements consensus. Clients propose commands by sending them to the protocol. The protocol eventually chooses a single one of the proposed values and returns it to the clients.

Paxos [12, 15] is one of the oldest and most well studied consensus protocols. We will see later that BPaxos uses Paxos to implement consensus, so it is important to be familiar with *what* Paxos is. Fortunately though, BPaxos treats Paxos like a black box, so we do not have to concern ourselves with *how* Paxos works.

2.2 MultiPaxos

Whereas consensus protocols like Paxos agree on a *single* value, **state machine replication** protocols like MultiPaxos agree on a *sequence* of values called a log. A state machine replication protocol involves some number of replicas of a state machine, with each state machine beginning in the same initial state. Clients propose commands to the replication

protocol, and the protocol orders the commands into an agreed upon log that grows over time. Replicas execute entries in the log in prefix order. By beginning in the same initial state and executing the same commands in the same order, all the replicas are guaranteed to remain in sync.

MultiPaxos [33] is one of the earliest and most popular state machine replication protocols. MultiPaxos uses one instance of Paxos for every log entry, agreeing on the log entries one by one. For example, it runs one instance of Paxos to agree on the command chosen in log entry 0, one instance for log entry 1, and so on. Over time, more and more commands are chosen, and the log of chosen commands grows and grows. MultiPaxos replicas execute commands as they are chosen, taking care not to execute the commands out of order.

For example, consider the example execution of a Multi-Paxos replica depicted in Figure 1. The replica implements a key-value store with keys *a* and *b*. First, the command $a \leftarrow 0$ (i.e. set *a* to 0) is chosen in log entry 0 (Figure 1a), and the replica executes the command (Figure 1b). Then, the command $a \leftarrow b$ is chosen in log entry 2 (Figure 1c). The replica cannot yet execute the command, because it must first execute the command in log entry 1, which has not yet been chosen (Figure 1d). Finally, $b \leftarrow 0$ is chosen in log entry 1 (Figure 1e), and the replica can execute the commands in both log entries 1 and 2. Note that the replica executes the log in prefix order, waiting to execute a command if previous commands have not yet been chosen and executed.



Figure 1: An example of a MultiPaxos replica executing commands over time, as they are chosen

MultiPaxos is implemented with a set of nodes called proposers and a set of nodes called acceptors. For this paper, we do not need to worry about the details of how MultiPaxos works, but let us focus briefly on its communication pattern. One of the proposers is designated a leader. Clients send all state machine commands to this single leader. When the leader receives a command *x*, it selects a log entry in which to place *x* and then performs one round trip of communication with the acceptors to get *x* chosen in the log entry. Then, it executes the command—once all commands in earlier log entries have been chosen and executed—and returns to the client. This communication pattern is illustrated in Figure 2.



Figure 2: MultiPaxos communication pattern. The leader is adorned with a crown.

2.3 Multileader and Generalized Consensus

MultiPaxos has a number of inefficiencies. Here, we focus on two well-known ones. First, MultiPaxos' throughput is bottlenecked by the leader. As shown in Figure 2, *every* command goes through the leader. Thus, MultiPaxos can run only as fast as the leader can. Protocols like Mencius [20], EPaxos [23], and Caesar [6] bypass the single leader bottleneck by having multiple leaders that can process requests in parallel. We call these protocols **multileader** protocols.

Second, MultiPaxos requires that replicas execute *all* commands in the same order. That is, MultiPaxos establishes a *total order* of commands. This is overkill. If two commands commute, they can be executed by replicas in either order. For example, key-value store replicas executing the log in Figure 1 could execute commands $a \leftarrow 0$ and $b \leftarrow 0$ in either order since the two commands commute. More formally, we say two commands **conflict** if executing them in opposite orders yields either different outputs or a different final state. State machine replication protocols that only require *conflicting* commands to be executed in the same order are said to implement generalized consensus [13]. Colloquially, we say such a protocol is **generalized**. Generalized protocols establish a *partial order* of commands (as opposed to a total order) in which only conflicting commands have to be ordered.

As a MultiPaxos leader receives commands from clients, it places them in increasing log entries. The first command is placed in log entry 0, the second in log entry 1, and so on. In this way, the leader acts as a sequencer, sequencing commands into a single total order. Multileader protocols however, by virtue of having multiple leaders, do not have a single designated node that processes every command. This makes it challenging to establish a single total order. As a result, most multileader protocols are also generalized. With multiple concurrently executing leaders, it is easier to establish a partial order than it is to establish a total order. Moreover, generalization allows leaders processing non-conflicting commands to operate completely independently from one another. While it is possible for a multileader protocol to establish a total order (e.g., Mencius [20]), such protocols run only as fast as the slowest replica (which lowers throughput), and involve all-to-all communication among the leaders (which also lowers throughput).

3 Bipartisan Paxos

BPaxos is a modular state machine replication protocol that is both multileader and generalized. Throughout the paper, we make the standard assumptions that the network is asynchronous, that state machines are deterministic, and that machines can fail by crashing but cannot act maliciously. We also assume that at most f machines can fail for some integervalued parameter f. Throughout the paper, we omit low-level protocol details involving the re-sending of dropped messages.

3.1 BPaxos Command Execution

MultiPaxos is *not* generalized. It totally orders all commands by sequencing them into a *log*. BPaxos is generalized, so it ditches the log and instead partially orders commands into a *directed graph*, like the ones shown in Figure 3.

BPaxos graphs are completely analogous to MultiPaxos logs. Every MultiPaxos log entry corresponds to a **vertex** in a BPaxos graph. Every MultiPaxos log entry holds a command; so does every vertex. Every log entry is uniquely identified by its index (e.g., 0); every vertex is uniquely identified by a **vertex id** (e.g., v_0). The one difference between graphs and logs are the edges. Every BPaxos vertex v has edges to some set of other vertices. These edges are called the **dependencies** of v. Note that we view a vertex's dependencies as belonging to the vertex, so when we refer to a vertex, we are also referring to its dependencies. The similarities between MultiPaxos logs and BPaxos graphs are summarized in Table 1.

Table 1: A comparison of MultiPaxos log entries and BPaxos vertices.

BPaxos	MultiPaxos	
graph	log	
vertex	log entry	
vertex id	index	
command	command	
dependencies	-	

MultiPaxos grows its *log* over time by repeatedly reaching consensus on one *log entry* at a time. BPaxos grows its *graph* over time by repeatedly reaching consensus on one *vertex* (and its dependencies) at a time. MultiPaxos replicas execute logs in prefix order, making sure not to execute a command until after executing *all previous commands*. BPaxos replicas execute graphs in prefix order (i.e. reverse topological order),





(f) $b \leftarrow 0$, $a \leftarrow b$ are executed.

Figure 3: An example of a BPaxos replica executing commands over time, as they are chosen.

making sure not to execute a command until after executing *its dependencies*.

An example of how BPaxos graphs grow over time and how a BPaxos replica executes these graphs in shown in Figure 3. As you read through the figure, note the similarities with Figure 1. First, the command $a \leftarrow 0$ is chosen in vertex v_0 with no dependencies (Figure 3a). Because the vertex has no dependencies, the replica executes $a \leftarrow 0$ immediately (Figure 3b). Next, the command $a \leftarrow b$ is chosen in vertex v_2 with dependencies on vertices v_0 and v_1 (Figure 3c). v_2 depends on v_1 , but a command has not yet been chosen in v_1 , so the replica does *not* yet execute $a \leftarrow b$ (Figure 3d). Finally, the command $b \leftarrow 0$ is chosen in vertex v_1 with no dependencies (Figure 3e). Because $b \leftarrow 0$ has no dependencies, the replica executes it immediately. Moreover, all of v_2 's dependencies have been executed, so the replica now executes $a \leftarrow b$ (Figure 3f).

Before we discuss the mechanisms that BPaxos uses to construct these graphs, note the following three graph properties.

Vertices are chosen once and for all. BPaxos reaches consensus on every vertex, so once a vertex has been chosen, it will never change. Its command will not change, it will not lose dependencies, and it will not get new dependencies.

Cycles can happen, but are not a problem. We will see in a moment that BPaxos graphs can sometimes be cyclic. These cycles are a nuisance, but easily handled. Instead of executing



Figure 4: An example of a BPaxos replica executing a cyclic graph. (a) y cannot be executed until v_z is chosen. (b) v_z is chosen. v_y and v_z form a strongly connected component. (c) y and z are executed in an arbitrary yet deterministic order; y then z or z then y.

graphs in reverse topological order one *command* at a time, replicas instead execute graphs in reverse topological order one *strongly connected component* at a time. The commands within a strongly connected component are executed in an arbitrary yet deterministic order (e.g., in vertex id order). This is illustrated in Figure 4.

Conflicting commands depend on each other. Because BPaxos is generalized, only conflicting commands have to be ordered with respect to each other. BPaxos ensures this by maintaining the following invariant:

Invariant 1 (dependency invariant). If two conflicting commands *x* and *y* are chosen in vertices v_x and v_y , then either v_x depends on v_y or v_y depends on v_x or both. That is, there is at least one edge between vertices v_x and v_y .

If two commands have an edge between them, every replica executes them in the same order. The dependency invariant ensures that every conflicting pair of commands has an edge between them, ensuring that all conflicting commands are executed in the same order. Non-conflicting commands do not need an edge between them and can be executed in any order.

3.2 Protocol Overview

BPaxos is composed of five modules: a dependency service, a consensus service, a set of leaders, a set of proposers, and a set of replicas. Here, we give an overview on how these modules interact by walking through the example execution shown in Figure 5. In the next couple of sections, we discuss each module in more detail.

1. A client *c* sends a state machine command *x* to leader l_0 . Note that all of the leaders process commands in parallel and that clients can send commands to any of them.

2. Upon receiving command x, l_0 generates a globally unique vertex id v_x for x. It then sends the message $\langle v_x, x \rangle$ to the dependency service.

3. Upon receiving message $\langle v_x, x \rangle$, the dependency service computes a set of dependencies deps_x for vertex v_x . Later, we will see exactly how the dependency service computes dependencies. For now, we overlook the details. The dependency service then sends back the message $\langle v_x, x, \text{deps}_x \rangle$ to l_0 .

4. l_0 forwards $\langle v_x, x, \text{deps}_x \rangle$ to proposer p_0 .

5. p_0 sends the message $\langle v_x, x, \text{deps}_x \rangle$ to the consensus service, proposing that the value (x, deps_x) be chosen in vertex v_x .

6. The consensus service implements one instance of consensus for every vertex. Upon receiving $\langle v_x, x, \text{deps}_x \rangle$, it chooses the value (x, deps_x) for vertex v_x and notifies p_0 with the message $\langle v_x, x, \text{deps}_x \rangle$. Note that in this example, the consensus service chose the value proposed by p_0 . In general, the consensus service may choose some other value if other proposers are concurrently proposing different values for vertex v_x . However, we will see later that this can only happen during recovery and is therefore not typical.

7. After p_0 learns that command x with dependencies deps_x has been chosen in vertex v_x , it notifies the replicas by broadcasting the message $\langle v_x, x, \text{deps}_x \rangle$.

8. Every replica manages a graph of chosen commands, as described in the previous subsection. Upon receiving $\langle v_x, x, \text{deps}_x \rangle$, a replica adds the vertex v_x to its graph with command x and dependencies deps_x . As described earlier, the replicas execute their graphs in reverse topological order. Once they have executed command x, yielding output o, one of the replicas sends back the response to the client c. Given r replicas, replica i sends back the response where $i = \text{hash}(v_x) \% r$ for some hash function.

Pseudocode for BPaxos is given in Figure 6, and a TLA+ specification of BPaxos is given in Appendix A. We now detail each BPaxos module. In the next section, we discuss why the dependency service, consensus service, and replicas do not scale and why the leaders and proposers do.

3.3 Dependency Service

When the dependency service receives a message of the form $\langle v_x, x \rangle$, it replies with a set of dependencies deps_x for v_x using the message $\langle v_x, x, \text{deps}_x \rangle$.

Concretely, we implement the dependency service with 2f + 1 dependency service nodes. Every dependency service node maintains a single piece of state, commands. commands is the set of all the messages that the dependency service node has received to date. When a dependency service node receives message $\langle v_x, x \rangle$ from a leader, it computes the dependencies of v_x as the set of all vertices v_y in commands that contain a command that conflicts with *x*:

deps = { v_y | $\langle v_y, y \rangle \in$ commands and *x* and *y* conflict}.

It then adds $\langle v_x, x \rangle$ to commands and sends $\langle v_x, x, deps \rangle$ back to the leader. When a leader sends a message



Figure 5: An overview of BPaxos execution. Note that we show the execution of only a single command for simplicity, so only one leader and one proposer are active. In a real BPaxos deployment, there are multiple clients and every leader and every proposer is active.

 $\langle v_x, x \rangle$ to the dependency service, it sends it to every dependency service node. Upon receiving f + 1 responses, $\{\langle v_x, x, \text{deps}_1 \rangle, \dots, \langle v_x, x, \text{deps}_{f+1} \rangle\}$, the leader computes the final dependencies as $\bigcup_{i=1}^{f+1} \text{deps}_i$, the union of the computed dependencies.

The dependency service maintains the following invariant.

Invariant 2 (dependency service invariant). If the dependency service produces responses $\langle v_x, x, \text{deps}_x \rangle$ and $\langle v_y, y, \text{deps}_y \rangle$ for conflicting commands *x* and *y*, then $v_x \in \text{deps}_y$ or $v_y \in \text{deps}_x$ or both.

That is, the dependency service computes dependencies such that conflicting commands depend on each other. Note that the dependency service invariant (Invariant 2) is very similar to the dependency invariant (Invariant 1). This is not an accident. Only dependencies computed by the dependency service can be chosen, so the dependency service invariant suffices to guarantee that the dependency invariant is maintained.

Theorem 1. The dependency service maintains Invariant 2.

Proof. Assume the dependency service produces responses $\langle v_x, x, \text{deps}_x \rangle$ and $\langle v_y, y, \text{deps}_y \rangle$ for conflicting commands x and y. We want to show that $v_x \in \text{deps}_y$ or $v_y \in \text{deps}_x$ or both. deps_x is the union of dependencies computed by some set Q_x of f + 1 dependency service nodes. Similarly, deps_y is the union of dependencies computed by some set Q_y of f + 1 dependency service nodes. Any two sets of f + 1 nodes must intersect (f + 1 is a majority of 2f + 1). Consider a dependency service node d in the intersection of Q_x and Q_y . d received both $\langle v_x, x \rangle$ and $\langle v_y, y \rangle$. Without loss of generality, assume it received $\langle v_y, y \rangle$ second. Then, when d received $\langle v_y, y \rangle$, $\langle v_x, x \rangle$ was already in its commands, so it must have included



Figure 6: BPaxos pseudocode

 v_x in its computed dependencies for v_y . deps_y is a union of dependencies that includes the dependencies computed by *d*. Thus, $v_x \in \text{deps}_y$. This is illustrated in Figure 7.



Figure 7: An illustration of the proof of Theorem 1.

Note that if the dependency service produces responses $\langle v_x, x, \text{deps}_x \rangle$ and $\langle v_y, y, \text{deps}_y \rangle$ for conflicting commands x and y, it may include $v_x \in \text{deps}_y$ and $v_y \in \text{deps}_x$. For example, if dependency service node d_1 receives x then y while dependency service node d_2 receives y then x, then dependencies formed from d_1 and d_2 will have v_x and v_y in each other's dependencies. This is the reason why BPaxos graphs may develop cycles.

Also note that the dependency service is an independent module within BPaxos. The dependency service is unaware of consensus, or BPaxos graphs, or state machines, or any other detail outside of the dependency service. The dependency service can be completely understood in isolation. In contrast, dependency computation in EPaxos and Caesar is tightly coupled with the rest of the protocol. For example, in Caesar, every command is assigned a timestamp. If a node receives two commands out of timestamp order, it must first wait to see if the higher timestamp command gets chosen with a dependency on the lower timestamp command before it is able to compute the lower timestamp command's dependencies. This coupling prevents us from understanding dependency computation in isolation.

3.4 Leaders

When a leader receives a command x from a client, it assigns x a globally unique vertex id v_x . The mechanism by which leaders generate unique ids is unimportant. You can use any mechanism you would like as long as ids are globally unique. In our implementation, a vertex id is a tuple of the leader's index and a monotonically increasing id beginning at 0. For example, leader 2 generates vertex ids (2,0), (2,1), (2,2), and so on.

After generating a vertex id v_x , the leader sends $\langle v_x, x \rangle$ to all dependency service nodes, aggregates the dependencies from

f + 1 of them, and forwards the dependencies to a proposer.

3.5 Proposers and Consensus Service

When a proposer receives a message $\langle v_x, x, \text{deps}_x \rangle$, it proposes to the consensus service that the value (x, deps_x) be chosen for vertex v_x . The consensus service implements one instance of consensus for every vertex, and eventually informs the proposer of the value (x', deps'_x) that was chosen for vertex v_x . In the normal case, (x', deps'_x) is equal to (x, deps_x) , but the consensus service is free to choose any value proposed for vertex v_x .

You can implement the consensus service with any consensus protocol that you would like. In our implementation of BPaxos, BPaxos proposers are Paxos proposers, and the consensus service is implemented as 2f + 1 Paxos acceptors. We implement Paxos with the standard optimization that phase 1 of the protocol can be skipped in round 0 (a.k.a. ballot 0). Doing so, and partitioning vertex ids uniformly across proposers, the proposers can get a value chosen in one round trip to the acceptors (in the common case). This optimization is very similar to the one done in MultiPaxos.

Again, note that the consensus service is an independent module that we can understand in isolation. The consensus service implements consensus, and that is it. It is unaware of dependencies, graphs, or any other detail of the protocol. Moreover, note that the consensus service is not specialized at all to BPaxos. We are able to use the Paxos protocol without modification. This lets us avoid having to prove a specialized implementation of consensus correct.

3.6 Replicas

Every BPaxos replica maintains a BPaxos graph and an instance of a state machine. Every state machine begins in the same initial state. Upon receiving a message $\langle v_x, x, \text{deps}_x \rangle$ from a proposer, a replica adds vertex v_x to its graph with command x and with edges to deps_x. As discussed earlier, the replicas execute their graphs in reverse topological order, one component at a time. When a replica is ready to execute a command x, it passes it to the state machine. The state machine transitions to a new state and produces some output o. One replica then returns o to the client that initially proposed x. In particular, given n replicas, r_i returns outputs to clients for vertices v_x where hash $(v_x) \% n = i$.

3.7 Summary

In summary, BPaxos is composed of five modules: leaders, dependency service nodes, proposers, a consensus service, and replicas. Clients propose commands; leaders assign unique ids to commands; the dependency service computes dependencies (ensuring that conflicting commands depend on each other); the proposers and consensus service reach consensus on every vertex; and replicas execute commands.

3.8 Fault Tolerance and Recovery

BPaxos can tolerate up to f failures. By deploying f + 1 leaders, proposers, and replicas, BPaxos guarantees that at least one of each is operational after f failures. The dependency service deploys 2f + 1 dependency service nodes, ensuring that at a quorum of f + 1 nodes is available despite f failures. The consensus service tolerates f failures by assumption. In our implementation, we use 2f + 1 Paxos acceptors, as is standard.

However, despite this, failures can still lead to liveness violations if we are not careful. A replica executes vertex v_x only after it has executed v_x 's dependencies. If one of v_x 's dependencies has not yet been chosen, then the execution of v_x is delayed. For example, in Figure 3, the execution of v_2 is delayed until after v_1 has been chosen and executed.

If a vertex v_x depends on a vertex v_y that remains forever unchosen, then v_x is never executed. This situation is rare, but possible in the event of failures. For example, if two leaders l_x and l_y concurrently send commands x and y in vertices v_x and v_y to the dependency service, and if l_y then crashes, it is possible that v_x gets chosen with a dependency on v_y , but v_y remains forever unchosen.

Dealing with these sorts of failure scenarios to ensure that every command eventually gets chosen is called **recovery**. Every state machine replication protocol has to implement some form of recovery, and for many protocols (though not all protocols), recovery is its most complicated part.

Fortunately, BPaxos' modularity leads to a very simple recovery protocol. When a replica notices that a vertex v_x has been blocked waiting for another vertex v_y for more than some configurable amount of time, the replica contacts the consensus service and proposes that a no-operation command noop be chosen for vertex v_y with no dependencies. noop is a special command that does not affect the state machine and does not conflict with any other command. Eventually, the consensus protocol returns the chosen value to the replica, and the execution of v_x can proceed.

4 Disaggregating and Scaling

BPaxos' modular design leads to high throughput in two ways: disaggregation and scaling.

4.1 Identifying Bottlenecks

The throughput of a protocol is determined by its bottleneck. Before we discuss BPaxos' throughput, we discuss how to identify the bottleneck of a protocol. Identifying a bottleneck with complete accuracy is hard. Protocol bottlenecks are affected by many factors including CPU speeds, network bandwidth, message sizes, workload characteristics, and so on. To make bottleneck analysis tractable, we make a major simplifying assumption. The assumption is best explained by way of an example.



Figure 8: MultiPaxos' throughput bottleneck. Every node is annotated with the number of messages that it sends and receives. MultiPaxos' throughput is proportional to $\frac{1}{2N+2}$.

Consider the execution of MultiPaxos shown in Figure 8 in which a client proposes a command *x*. The execution involves $N \ge 2f + 1$ acceptors. We have annotated each node with the number of messages it sends and receives in the process of handling *x*. The leader p_0 processes 2N + 2 messages, and every acceptor processes 2 messages. Our major assumption is that the time required for each node to process command *x* is directly proportional to the number of messages that it processes. Thus, the leader takes time proportional to 2N + 2, and the acceptors take time proportional to 2. This means that the leader is the bottleneck, and the protocol's throughput is directly proportional to $\frac{1}{2N+2}$, the inverse of the time required by the bottleneck component.

While our assumption is simplistic, we will see in Section 6 that empirically it is accurate enough for us to identify the actual bottleneck of protocols in practice. Now, we turn our attention to BPaxos. Consider the execution of BPaxos shown in Figure 9. We have $N \ge 2f + 1$ dependency service nodes, N acceptors, $L \ge f + 1$ leaders, L proposers, and $R \ge f + 1$ replicas¹.

Again, we annotate each node with the number of messages it processes to handle the client's command. The dependency service nodes and acceptors process two messages each. The replicas process either one or two messages—depending on whether they are returning a response to the client—for an average of $1 + \frac{1}{R}$. The leaders and proposers process significantly more messages, 2N + 2 and 2N + R + 1 messages respectively. Thus, the throughput through a *single* leader and proposer is proportional to $\frac{1}{2N+R+1}$. Unlike MultiPaxos though, BPaxos does not have a single leader. All *L* of the leaders and proposers execute concurrently, with client commands divided amongst them. With *L* leaders and proposers, BPaxos' throughput is proportional to $\frac{L}{2N+R+1}$.



Figure 9: BPaxos' throughput bottleneck. Every node is annotated with the number of messages that it sends and receives. BPaxos' throughput is proportional to $\frac{L}{2N+R+1}$.

4.2 Disaggregation

Many state machine replication protocols pack multiple logical nodes onto a single physical node. We could do something similar. We could deploy N = L = R dependency service nodes, acceptors, leaders, proposers, and replicas across N physical "super nodes", with one of each component colocated on a single physical machine. This would reduce the latency of the protocol by two network delays and open the door for optimizations that could reduce the latency even further.

However, aggregating logical components together would worsen our bottleneck. Now, for a given command, a super node would have to process the messages of a dependency service node, an acceptor, a leader, a proposer, and a replica. With the bottleneck component processing more messages per command, the throughput of the protocol decreases. Disaggregating the components allows for pipeline parallelism in which load is more evenly balanced across the components.

4.3 Scaling

Scaling is a classic systems technique that is used to increase the throughput of a system. However, to date, consensus protocols have not been able to take full advantage of scaling. Conventional wisdom for replication protocols suggests that we use as few nodes as possible. Returning to Figure 8, we see this conventional wisdom in action. The throughput of Multi-Paxos is proportional to $\frac{1}{2N+2}$. Adding more proposers does not do anything, and adding more acceptors (i.e. increasing *N*) *lowers* the throughput.

BPaxos revises conventional wisdom and notes that while some components are hard or impossible to scale (e.g., ac-

¹We can have a different number of leaders and proposers, but letting them be equal simplifies the example.

ceptors), other components scale trivially. Serendipitously, the components that are easy to scale turn out to be the same components that are a throughput bottleneck.

More specifically, we learned from Figure 9 that BPaxos' throughput is proportional to $\frac{L}{2N+R+1}$ with the *L* leaders and proposers being the bottleneck. To increase BPaxos' throughput, we simply increase *L*. We can increase the number of leaders and proposers until they are no longer the bottleneck. This pushes the bottleneck to either the dependency service nodes, the acceptors, or the replicas. Fortunately, these nodes only process at most two messages per command. This is equivalent to an unreplicated state machine which must at least receive and execute a command and reply with the result. Thus, we have effectively shrunk the throughput bottleneck to its limit.

Note that we are able to perform this straightforward scaling because BPaxos' components are modular. When we co-locate components together, L = N = R, and it is impossible for us to increase L (which increases throughput) without increasing N and R (which decreases throughput). Modularity allows us to scale each component independently.

5 Practical Considerations

5.1 Ensuring Exactly Once Semantics

If a client proposes a command to a state machine replication protocol but does not hear back quickly enough, it resends the command to the protocol to make sure that the command eventually gets executed. Thus, a replication protocol might *receive* a command more than once, but it has to guarantee that it never *executes* the command more than once. Executing a command more than once would violate exactly once semantics.

Non-generalized protocols like Paxos [33], Viewstamped Replication [18], and Raft [25] all employ the following technique to avoid executing a command more than once. First, before a client proposes a command to a replication protocol, it annotates the command with a monotonically increasing integer-valued id. Moreover, clients only send one command at a time, waiting to receive a response from one command before sending another. Second, every replica maintains a client table, like the one illustrated below. A client table has one entry per client. The entry for a client records the largest id of any command that the replica has executed for that client, along with the result of executing the command with that id. A replica only executes commands for a client if it has a larger id than the one recorded in the client table. If it receives a command with the same id as the one in the client table, it replies with the recorded output instead of executing the command a second time.

Client	Id	Output
10.31.14.41	2	"foo"
10.54.13.123	1	"bar"

Naively applying this same trick to BPaxos (or any generalized protocol) is unsafe. For example, imagine a client issues command x with id 1. The command gets chosen and is executed by replica 1. Then, the client issues non-conflicting command y with id 2. The command gets chosen and is executed by replica 2. Because y has a larger id than x, replica 2 will never execute x.

To fix this bug, a replica must record the ids of *all* commands that it has executed for a client, along with the output corresponding to the largest of these ids. Replicas only execute commands they have not previously executed, and relay the cached output if they receive a command with the corresponding id.

5.2 Dependency Compaction

Upon receiving a command x in vertex v_x , a dependency service node returns the set of all previously received vertices with commands that conflict with x. Over time, as the dependency service receives more and more commands, these dependency sets get bigger and bigger. As the dependency sets get bigger, BPaxos' throughput decreases because more time is spent sending these large dependency sets, and less time is spent doing useful work.

To combat this, a BPaxos dependency service node has to compact dependencies in some way. Recall that BPaxos leader *i* creates vertex ids (i,0), (i,1), (i,2), and so on. Thus, vertex ids across all the leaders form a two-dimensional array with one column for every leader index and one row for every monotonically increasing id.



Figure 10: An example of dependency compaction

For example, consider a dependency service node that has received commands a, b, c, d, and e in vertices (0, 1), (0, 0), (1, 2), (1, 0), and (2, 1) as shown in Figure 10. Without dependency compaction, if the dependency service node receives a command that conflicts with commands a, b, c, d, and e, it would return the vertex ids of these five commands. In our example, the dependency service node returns only five dependencies, but in a real deployment, the node could return hundreds of thousands of dependencies.

With dependency compaction on the other hand, the dependency service node instead artificially adds more dependencies. In particular, for every leader i, it computes the largest id j for which a dependency (i, j) exists. Then, it adds

 $\{(i,k) | k \le j\}$ to the dependencies. In other words, it finds the largest dependency in each column and then adds all of the vertex ids below it as dependencies. In Figure 10, the inflated set of dependencies is highlighted in blue. Even though more dependencies have been added, the set of inflated dependencies can be represented more compactly, with a single integer for every leader (i.e., the id of the largest command for that leader). Thus, every BPaxos dependency set can be succinctly represented with *N* integers (for *N* leaders).

6 Evaluation

6.1 Latency and Throughput

Experiment Description. We implemented MultiPaxos, EPaxos², and BPaxos in Scala³. Here, we measure the throughput and latency of the three protocols with respect to three parameters: the number of clients, the conflict rate, and the parameter f.

- Clients. Clients propose commands in a closed loop. That is, after a client proposes a command, it waits to receive a response before proposing another command. We also run multiple clients in the same process, so deployments with a large number of clients (e.g., 1200 clients) may use only a few client processes. We run 1, 10, 50, 100, 300, 600, and 1200 clients.
- Conflict rate. The protocols replicate a key-value store state machine. Commands are single key gets or single key sets. With a conflict rate of r, r of the commands are sets to a single key, while (1 r) of the commands are gets to other keys. Keys and values are both eight bytes. If commands are large, the data path and control path can be split, as in [8]. We run with r = 0, r = 0.02, and r = 0.1. As described in [23], workloads in practice often have very low conflict rates.
- *f*. Recall that a protocol with parameter *f* must tolerate at most *f* failures. We run with f = 1 and f = 2.

We deploy the three protocols on m5.4xlarge EC2 instances within a single availability zone. MultiPaxos deploys f + 1 proposers and 2f + 1 acceptors. EPaxos deploys 2f + 1replicas. BPaxos deploys 2f + 1 dependency service nodes, 2f + 1 acceptors, f + 1 replicas, 5 leaders and proposers when f = 1, and 10 leaders and proposers when f = 2. Every logical node is deployed on its own physical machine, except that every BPaxos leader is co-located with a BPaxos proposer. The protocols do not perform batching. All three protocols implement thriftiness, a standard optimization [23].

Results. The benchmark results are shown in Figure 11. In Figure 11a with f = 1, we see that MultiPaxos achieves a peak throughput of roughly 25,000 to 30,000 commands per second. EPaxos achieves a peak throughput of 30,000 to 40,000 depending on the conflict rate. BPaxos achieves 70,000 to 75,000, nearly double that of EPaxos. Both EPaxos' and BPaxos' throughput decrease with higher conflict rate. Higher conflict rates lead to graphs with more edges, which increases the time required to topologically sort the graphs.

Note that the EPaxos implementation in [23] achieves a peak throughput of 45,000 to 50,000, slightly higher than our implementation. We believe the discrepancy is due to implementation language (Go vs Scala) and various optimizations performed in [23] that we have not implemented (e.g., a custom marshaling and RPC compiler [3]). We believe that if we apply the same optimizations to our implementations, all three protocols' throughput would increase similarly.

In Figure 11b, with f = 2, MultiPaxos' peak throughput has decreased to 20,000, EPaxos' peak throughput has decreased to 25,000, and BPaxos' peak throughput has decreased to 65,000. As f increases, the MultiPaxos leader has to contact more nodes, so the drop in throughput is expected. With f = 2, EPaxos and BPaxos both have more leaders. More leaders increases the likelihood of cycles, which slows the protocols down slightly. Moreover, when performing dependency compaction as described in Section 5, the number of dependencies scales with the number of leaders. BPaxos's peak throughput is still roughly double that of EPaxos.

After sending a command, a BPaxos client must wait eight network delays to receive a response. MultiPaxos and EPaxos require only four. Thus, under low load, MultiPaxos and EPaxos have lower latency than BPaxos. In Figure 11c, we see that with a single client, MultiPaxos and EPaxos have a latency of roughly 0.25 ms, whereas BPaxos has a latency of 0.41. Under high load though, BPaxos achieves lower latency. With 10 clients, the latency of the three protocols is roughly even, and with 50 clients, BPaxos's latency has already dropped below that of the other two protocols. In Figure 11a and Figure 11b, we see that under higher loads of 600 and 1200 clients, BPaxos's latency can be two to six times lower than the other two protocols.

Note that our results are specific to our deployment within a single data center. With a geo-replicated deployment, MultiPaxos and EPaxos would both outperform BPaxos. In this scenario, minimizing network delays is essential for high performance. Also note that BPaxos uses more machines than MultiPaxos and EPaxos in order to achieve higher throughput via disaggregation and scaling. This makes BPaxos a poor fit in resource constrained environments.

²Note that we implement *Basic* EPaxos, the algorithm outlined in [22]. In general, Basic EPaxos has larger quorums and simpler recovery compared to the complete EPaxos protocol which is described in [23]. For f = 1 though, the performance of the two protocols is practically identical.

³To mitigate the effects of JVM garbage collection on our experiments, we run our experiments with a large heap size of 32GB and run experiments for only a short amount of time.



(a) Latency-throughput curves for Multipaxos, EPaxos, and BPaxos. EPaxos and BPaxos are run with 0%, 2% and 10% conflict rates. Here, f = 1.

(b) The same as Figure 11a but with f = 2.

(c) Median latency values (ms) from Figure 11a.





Figure 12: An ablation study showing the effect of disaggregation and scaling on throughput and latency with 600 clients and one client. Throughput for one client is not shown because it is simply the inverse of latency.

6.2 Ablation Study

Experiment Description. The previous experiment showed that BPaxos can achieve roughly double the throughput of EPaxos. Now, we analyze how BPaxos achieves these speedups. In particular, we perform an ablation study to measure how BPaxos' disaggregation and scaling affect its throughput. We repeat the experiment from above with f = 1, with r = 0, and with 1 and 600 clients. We vary the number of leaders from 3 to 7. Moreover, we also consider a "coupled BPaxos" deployment with three machines where each machine runs a single process that acts as a leader, a dependency service node, a proposer, an acceptor, and a replica. This artificially coupled BPaxos is similar to EPaxos in which every replica plays many roles.

Results. The results of the experiment are shown in Figure 12. In Figure 12a, we see the throughput of the coupled BPaxos deployment is only 20,000 under high load. This is

lower than both MultiPaxos and EPaxos. When we decouple the protocol and run with three leaders, the throughput increases threefold to 60,000. Disaggregating the nodes introduces pipeline parallelism and reduces the load on the bottleneck component. As we increase to five leaders, the throughput increases to a peak of 75,000. At this point, the leaders are not the bottleneck and adding more leaders only serves to slow down the protocol (for reasons similar to why the f = 2 deployment of BPaxos is slightly slower than the f = 1 deployment).

In Figure 12b, we see that the coupled protocol has roughly six times the latency compared to the decoupled protocol under high load. Moreover, the number of leaders doesn't have much of an impact on the latency. In Figure 12c, we see that the coupled protocol has lower latency compared to the decoupled protocol under low load, as fewer messages have to traverse the network. These results are consistent with the previous experiment. Coupled protocols can achieve lower latency under low load but decoupled protocols achieve higher throughput and lower latency under high load.

In summary, both disaggregation and scaling contribute significantly to BPaxos' increased throughput and lower latency under high load, and they also explain why BPaxos has higher latency under low load.

6.3 Batching

Existing state machine replication protocols can perform batching to increase their throughput at the cost of some latency [22, 28, 30]. BPaxos uses decoupling and scaling to increase throughput at the cost of some latency. These two techniques accomplish the same goal but are orthogonal. We can add batching to BPaxos to increase its throughput even further. BPaxos leaders can collect batches of commands from clients and place all of them within a single vertex. While batching improves the throughput of all replication protocols, BPaxos' modular design enables the protocol to take advantage of batching particularly well.

First, the overheads of receiving client messages and forming batches falls onto the leaders. Because we can scale the leaders, these overheads can be amortized until they are no longer a bottleneck. Moreover, the execution time of proposers and acceptors increases linearly with the number of batches, not the number of commands. Thus, increasing the batch size also amortizes their overheads. Finally, as batch sizes grow, the number of vertices and edges in the replicas' graphs shrinks. Thus, replicas can topologically sort the smaller graphs faster.

We repeated the benchmarks from above with f = 1 and r = 0 with a batch size of 1000 and achieved a peak throughput of roughly 500,000 commands per second with a median latency of roughly 200 ms.

7 Related Work

Paxos, VR, Raft MultiPaxos [12, 15, 16, 21, 33], Raft [21], and Viewstamped Replication [18] are all single leader, nongeneralized state machine replication protocols. BPaxos has higher throughput than these protocols because it is not bottlenecked by a single leader. These protocols, however, have lower latency than BPaxos under low load and are much simpler.

Mencius Mencius [20] is a multi-leader, non-generalized protocol in which MultiPaxos log entries are round-robin partitioned among a set of leaders. Because Mencius is not generalized, a log entry cannot be executed until *all* previous log entries have been executed. To ensure log entries are being filled in appropriately, Mencius leaders perform all-to-all communication between each other. This prevents leaders from scaling and prevents other throughput-improving optimizations such as thriftiness.

Generalized GPaxos Generalized Paxos [13] and GPaxos [31] are generalized, but not fully multi-leader. Clients can send commands directly to acceptors, behaving very much like a leader. However, in the face of collisions, Generalized Paxos and GPaxos rely on a single leader to resolve the collision. This single leader becomes a bottleneck in high contention workloads and prevents scaling.

EPaxos and Caesar EPaxos [22,23], like BPaxos, is generalized and multi-leader. EPaxos has lower latency than BPaxos (four network delays as opposed to eight). EPaxos is a tightly coupled protocol. Every node acts as a leader, dependency service node, proposer, acceptor, and replica. This increases the load on the bottleneck nodes and also prevents disaggregation and scaling. EPaxos, like Fast Paxos, optimistically takes a "fast path" before sometimes reverting to a "slow path". This allows the protocol to execute a command in four network delays in the best case, but fast paths significantly complicate the protocol. For example, recovery in the face of fast paths can deadlock if not implemented correctly. Caesar [6] is very similar to EPaxos, with slight tweaks that increase the odds of the fast path being taken.

A Family of Leaderless Generalized Algorithms In [19], Losa et al. present a generic architecture for leaderless (what we call multi-leader) generalized consensus protocols. The generic algorithm is very similar to BPaxos. In fact, some parts like the dependency service are practically identical. However, the three page paper does not present any implementations and focuses more on the theory behind abstracting the commonalities shared by existing leaderless generalized algorithms. BPaxos fleshes out the design and improves on the work by discussing disaggregation, scaling, and practical considerations like ensuring exactly once semantics and dependency compaction.

Multi-Core Paxos In [29], Santos et al. describe how to increase the throughput of a single MultiPaxos node by decomposing the node into multiple components, with each component run on a separate core (e.g., one core for sending messages, one for receiving messages, and so on). This work complements BPaxos nicely. Santos et al. perform finegrained decoupling to improve the throughput of a single node, and BPaxos performs higher-level protocol decoupling to improve the throughput of the entire protocol.

SpecPaxos, NOPaxos, CURP SpecPaxos [27], NOPaxos [17], and CURP [26] all perform speculative execution to reduce latencies as low as two network delays. However, speculative execution on the fast path significantly increases the complexity of the protocols, and none of the protocols focus on disaggregation or scaling as a means to increase throughput.

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A BPaxos TLA+ Specification

```
----- MODULE SimpleBPaxos -----
(* This is a specification of Simple BPaxos. To keep things simple and to
                                                                  *)
(* make models more easily checkable, we abstract a way a lot of the
                                                                  *)
(* unimportant details of Simple BPaxos. In particular, the specification
                                                                  *)
(* does not model messages being sent between components and does not
                                                                  *)
(* include leaders, proposers, or replicas. The consensus service is also
                                                                  *)
(* left abstract. The core of Simple BPaxos is that dependency service
                                                                  *)
(* responses (noops) are proposed to a consensus service. This core of the
                                                                  *)
(* algorithm is what is modelled.
                                                                  *)
(*
                                                                  *)
(* Run 'tlc SimpleBPaxosModel' to check the model.
                                                                  *)
EXTENDS Dict, Integers, FiniteSets
*)
(* Constants
\ The set of commands that can be proposed to BPaxos. In this specification,
\* every command can be proposed at most once. This is mostly to keep behaviors
\* finite. In a real execution of Simple BPaxos, a command can be proposed an
\* infinite number of times.
CONSTANT Command
ASSUME IsFiniteSet (Command)
\* The command conflict relation. Conflict is a symmetric relation over Command
\ such that two commands a and b conflict if (a, b) is in Conflict.
CONSTANT Conflict
ASSUME
   /\ Conflict \subseteq Command \X Command
   /\ \A ab \in Conflict : <<ab[2], ab[1]>> \in Conflict
\* We assume the existence of a special noop command that does not conflict
\* with any other command. Because noop is not in Command, it does not appear
\* in Conflict.
CONSTANT noop
ASSUME noop \notin Command
\* The set of dependency service nodes.
CONSTANT DepServiceNode
ASSUME IsFiniteSet (DepServiceNode)
\* The set of dependency service quorums. Every two quorums must interesct.
\* Typically, we deploy 2f + 1 dependency service replicas and let quorums be
\ sets of replicas of size f + 1.
CONSTANT DepServiceQuorum
ASSUME
   // \A Q \in DepServiceQuorum : Q \subseteq DepServiceNode
   /\ \A Q1, Q2 \in DepServiceQuorum : Q1 \intersect Q2 /= {}
```

```
(* Variables and definitions.
                                                                         *)
\uparrow In Simple BPaxos, vertex ids are of the form Q.i where Q is a leader and i is
\* a monotonically increasing id (intially zero). In this specification, we
\* don't even model Simple BPaxos nodes. So, we let instances be simple
\* integers. You might imagine we would say 'VertexId == Nat', but keeping
\uparrow things finite helps TLC. Every command can be proposed at most once, so
\* allowing instances to range between 0 and |Command| works great.
VertexId == 0..Cardinality(Command)
\* A proposal is a command (or noop) and its dependencies.
Proposal == [cmd: Command \union {noop}, deps: SUBSET VertexId]
\* The proposal associated with noop. Noop doesn't conflict with any other
\* command, so its dependencies are always empty.
noopProposal == [cmd |-> noop, deps |-> {}]
\* A dependency graph is a directed graph where each vertex is labelled with an
\* vertex id and contains a command. We model the graph as a dictionary mapping
\* a vertex id to its command and dependencies.
DependencyGraph == Dict(VertexId, Proposal)
\* dependencyGraphs[d] is the dependency graph maintained on dependency
\* service node d.
VARIABLE dependencyGraphs
\* The next vertex id to assign to a proposed command. It is initially 0 and
\* incremented after every proposed command.
VARIABLE nextVertexId
\* A dictionary mapping vertex id to the command proposed with that vertex id.
VARIABLE proposedCommands
\* A dictionary mapping vertex id to the set of proposals proposed to the
\* consensus service in that instance.
VARIABLE proposals
\* A dictionary mapping vertex id to the proposal that was chosen by the
\* consensus service for that vertex id.
VARIABLE chosen
vars == <<
 dependencyGraphs,
 nextVertexId,
 proposedCommands,
 proposals,
 chosen
>>
TvpeOk ==
  /\ dependencyGraphs \in Dict(DepServiceNode, DependencyGraph)
```

```
/\ nextVertexId \in VertexId
  /\ proposedCommands \in Dict(VertexId, Command)
  /\ proposals \in Dict(VertexId, SUBSET Proposal)
  /\ chosen \in Dict(VertexId, Proposal)
  _____
(* Actions.
                                                                       *)
\* Propose a command `cmd` to Simple BPaxos. In a real implementation of Simple
\* BPaxos, a client would send the command to a leader, and the leader would
\* forward the command to the set of dependency service nodes. Here, we bypass
\ all that. The only thing to do here is to assign the command an instance and
\* make sure it hasn't already been proposed.
ProposeCommand(cmd) ==
  /\ cmd \notin Values(proposedCommands)
 /\ proposedCommands' = [proposedCommands EXCEPT ![nextVertexId] = cmd]
 /\ nextVertexId' = nextVertexId + 1
 /\ UNCHANGED <<dependencyGraphs, proposals, chosen>>
\ Given a dependency graph G and command cmd, return the set of vertices in G
\* that contain commands that conflict with cmd. For example, consider the
\uparrow following dependency graph with commands b, c, and d in vertices v_b, v_c,
\ and v_d. If command a conflicts with c and d, then the dependencies of a are
\ v_c and v_d.
\ *
\*
                                v_b
                                    V C
                               +---+ +---+
\*
\ *
                               | b +---> c |
\ *
                               +-+-+
                                     +---+
\ *
                                \ *
                               +-17-+
\ *
                               | d |
\ *
                               +---+
\ *
                               v d
Dependencies(G, cmd) ==
  {v \in VertexId : G[v] /= NULL /\ <<cmd, G[v].cmd>> \in Conflict}
\* Here, dependency service node d processes a request in vertex v. Namely,
\* it adds v to its dependency graph (along with the command in
\* proposedCommands). Dependency service nodes also do not process a command
\* more than once. In a real Simple BPaxos implementation, the dependency
\ service node would receive a message from a leader and send dependencies
\* back to the leader. Also, a dependency service node could receive a request
\* from the leader more than once. We abstract all of this away.
DepServiceProcess(d, v) ==
 LET G == dependencyGraphs[d] IN
 /\ proposedCommands[v] /= NULL
 / \setminus G[v] = NULL
  /\ LET cmd == proposedCommands[v] IN
   /\ dependencyGraphs' = [dependencyGraphs EXCEPT ![d][v] =
                           [cmd |-> cmd, deps |-> Dependencies(G, cmd)]]
```

```
/\ UNCHANGED <<nextVertexId, proposedCommands, proposals, chosen>>
\* Evalutes to whether a quorum of dependency service nodes have processed the
\ command in vertex v.
ExistsQuorumReply(Q, v) ==
 \A d \in Q : dependencyGraphs[d][v] /= NULL
\* Evaluates to the dependency service reply for vertex v from quorum Q of
\* dependency service nodes.
QuorumReply(Q, v) ==
 LET responses == {dependencyGraphs[d][v] : d \in Q} IN
  [cmd |-> (CHOOSE response \in responses : TRUE).cmd,
  deps |-> UNION {response.deps : response \in responses}]
\* Propose a noop gadget in vertex v to the consensus service. In a real
\* Simple BPaxos implementation, a proposer would propose a noop only
\uparrow in some circumstances. In this model, we allow noops to be proposed at any
\* time.
ConsensusProposeNoop(v) ==
 /\ proposals' = [proposals EXCEPT ![v] = @ \union {noopProposal}]
 /\ UNCHANGED <<dependencyGraphs, nextVertexId, proposedCommands, chosen>>
\* Propose a dependency service reply in vertex v to the consensus service.
ConsensusPropose(v) ==
 \E Q \in DepServiceQuorum :
   /\ ExistsQuorumReply(Q, v)
   /\ proposals' = [proposals EXCEPT ![v] = @ \union {QuorumReply(Q, v)}]
   /\ UNCHANGED <<dependencyGraphs, nextVertexId, proposedCommands, chosen>>
\ Choose a value for vertex v.
ConsensusChoose(v) ==
 / \ proposals[v] /= \{ \}
 / \ chosen[v] = NULL
 /\ chosen' = [chosen EXCEPT ![v] = CHOOSE g \in proposals[v] : TRUE]
 /\ UNCHANGED <<dependencyGraphs, nextVertexId, proposedCommands, proposals>>
 _____
(* Specification.
                                                                       *)
Tnit ==
 /\ dependencyGraphs = [d \in DepServiceNode |-> [v \in VertexId |-> NULL]]
 /  nextVertexId = 0
 /\ proposedCommands = [v \in VertexId |-> NULL]
 /\ proposals = [v \in VertexId |-> {}]
 // chosen = [v \in VertexId |-> NULL]
Next ==
 \/ \E cmd \in Command : ProposeCommand(cmd)
 \/ \E d \in DepServiceNode : \E v \in VertexId : DepServiceProcess(d, v)
 \/ \E v \in VertexId : ConsensusProposeNoop(v)
 \/ \E v \in VertexId : ConsensusPropose(v)
 \/ \E v \in VertexId : ConsensusChoose(v)
```

```
FairSpec == Spec /\ WF_vars(Next)
                       _____
(* Properties and Invariants.
                                                                     *)
\* The consensus service can choose at most command in any given instance.
ConsensusConsistency ==
 A v \in V
   chosen[v] /= NULL => chosen'[v] = chosen[v]
AlwaysConsensusConsistency ==
 [][ConsensusConsistency]_vars
\uparrow If two conflicting commands a and b yield dependencies deps(a) and deps(b)
\uparrow from the dependency service, then a is in deps(b), or b is in deps(a), or
\ \ both.
DepServiceConflicts ==
 \A v1, v2 \in VertexId :
 \A Q1, Q2 \in DepServiceQuorum :
 IF v1 /= v2 /\ ExistsQuorumReply(Q1, v1) /\ ExistsQuorumReply(Q2, v2) THEN
    LET proposal1 == QuorumReply(Q1, v1)
       proposal2 == QuorumReply(Q2, v2) IN
    <<pre><<pre>proposal1.cmd, proposal2.cmd>> \in Conflict =>
      v1 \in proposal2.deps \/ v2 \in proposal1.deps
 ELSE
   TRUE
\* Simple BPaxos should only choose proposed commands. This is inspired by [1].
\ *
\* [1]: github.com/efficient/epaxos/blob/master/tla+/EgalitarianPaxos.tla
Nontriviality ==
 A v \in V
   chosen[v] /= NULL =>
     \/ chosen[v].cmd \in Values(proposedCommands)
     \ If two conflicting commands a and b are chosen, then a is in deps(b), or b
\ is in deps(a), or both.
ChosenConflicts ==
 \A v1, v2 \in VertexId :
 IF v1 /= v2 /\ chosen[v1] /= NULL /\ chosen[v2] /= NULL THEN
   LET proposal1 == chosen[v1]
      proposal2 == chosen[v2] IN
    <<pre><<pre>proposal1.cmd, proposal2.cmd>> \in Conflict =>
      v1 \in proposal2.deps \/ v2 \in proposal1.deps
 ELSE
   TRUE
\* True if every command is chosen.
```

Spec == Init /\ [][Next]_vars

```
EverythingChosen ==
 \A cmd \in Command :
   E v \in V VertexId :
     /\ chosen[v] /= NULL
     / \ chosen[v] = cmd
\* Fairness free theorem.
THEOREM
 Spec => /\ AlwaysConsensusConsistency
        /\ []DepServiceConflicts
        /\ []Nontriviality
        /\ []ChosenConflicts
\* True if no noops are chosen.
NoNoop ==
 ~ E v \in V
   /\ chosen[v] /= NULL
   (\land chosen[v].cmd = noop
\* If no noops are chosen, then every command is chosen. This property is only
\* true for FairSpec.
NoNoopEverythingChosen ==
 []NoNoop => <>EverythingChosen
\* Fairness theorem.
THEOREM
 FairSpec => /\ AlwaysConsensusConsistency
            /\ []DepServiceConflicts
            /\ []Nontriviality
            /\ []ChosenConflicts
            /\ NoNoopEverythingChosen
_____
----- MODULE Dict -----
(* TLA+ has the notion of functions. For example [A \rightarrow B] is the set of all
                                                                   *)
(* functions from the set A to the set B. Functions are a lot like the
                                                                   *)
(* dictionaries you find in a language like Python, except for one notable
                                                                   *)
(* distinction. A function f \in [A \to B] is total, so every value a \in A
                                                                   *)
(* must map to some value b \in B by way of f. Dictionaries from A to B, on
                                                                   *)
(* the other hand, do not have to map every a \in A to some corresponding b
                                                                   *)
(* \in B. This module builds up dictionaries out of functions. Doing so is
                                                                   *)
(* relatively straightforward. We introduce a NULL value and model a
                                                                   *)
(* Dictionary as a function [A \to B \cup {NULL}].
                                                                   *)
CONSTANT NULL
Dict(K, V) == [K \rightarrow V \setminus cup \{NULL\}]
Keys(dict) == {k \in DOMAIN dict : dict[k] /= NULL}
```

```
Values(dict) == {dict[k] : k \in Keys(dict)}
Items(dict) == {<<k, dict[k]>> : k \in Keys(dict)}
```